Understanding Eddy Currents in Conducting Discs: Factors & Analysis

Physics

A cylindrical region contains a homogenous, time-varying magnetic field $$\vec{B} = B_0 \cos(\omega t) \, \hat{a}_z$$; B_0 is the field magnitude and $$\hat{a}_z \, \vec{}$$ is vertical unit vector. Since there is no electric charge in the system, the changing magnetic field is the sole cause of the electric field $$\vec{E}$$. According to Farady's equation :

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$
(eq.1)

or in the case where $$\vec{B}$$ only has z component:

$$\frac{1}{r} \left(\begin{array}{c} \frac{\partial}{\partial r} \left(r E_{\phi} \right) - \frac{\partial E_r}{\partial \phi} \end{array} \right) \hat{a}_z = \omega B_0 \sin(\omega t) \hat{a}_z$$
(eq.2)

where r and $$\phi$$ represent radial and angular coordinates of the cylindrical coordinate system (Fig. 1).

Due to the symmetry of the problem, its analysis can be simplified by noting that $$E_r$$ does not depend on $$\phi$$, i.e. $$\frac{\partial E_r}{\partial \phi} = 0$$

$$
$$
. Equation 2 therefore, takes the following form:

$$
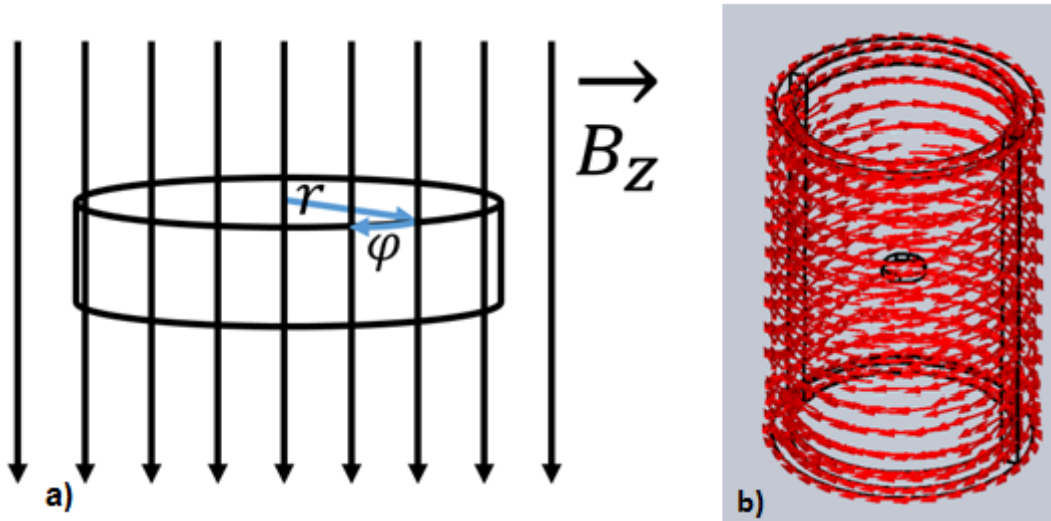\frac{1}{r} \frac{\partial}{\partial r} \left(r E_{\phi} \right) = \omega B_0 \sin(\omega t)
$$

(eq.3)

Figure 1 - a) *Copper disc in vertical magnetic field;*b) *Changing magnetic field is produced inside a cylinder wrapped in coils that carry AC current;*

To solve for r1??r??(rE??)=?B0?sin(?t)
, equation 3 is multiplied by ?E
and then integrated from 0 to ?=1
as:

rE??=?0r??B0?sin(?t)rdr

which leads to

$$
E_{\phi} = \frac{1}{2} \omega B_0 r \sin(\omega t)
$$

or E=21??B0?rsin(?t)a^??

If a copper disc is placed inside the cylinder, as a consequence of the induced electric field, eddy currents are distributed inside the disc. Current density j=21???B0?rsin(?t)a^??
is, according to the Ohm's law :

$$
\mathbf{j} \, \overset{\rightarrow}{=} \sigma \mathbf{E} \, \overset{\rightarrow}{=} \frac{1}{2} \omega \sigma B_0 r \sin(\omega t) \, \hat{a}_\phi
$$

where j=??E=?21???B0?rsin(?t)a^??
is specific conductivity of copper (?=5.7×107mS?). For a magnetic field with a magnitude of B0?=1.58×10?2T
and angular frequency ?=2??60rad/s
, magnitude of current density is j=r×1.69759×108m3A?
. Induced eddy currents lag the change in flux density by 90°.

Model

Eddy current distribution in a copper disc can be easily simulated in **EMS** as a **AC Magnetic**

study. To create a uniform magnetic field inside the cylinder, allow a certain thickness to its wall so that you can define the wall as a wound coil. A cylinder with inner radius of 15mm and height of 50mm, should be used to define a **Wound Coil** with 100 turns and **RMS current magnitude per turn** of $$\dfrac{10}{\sqrt{2}}$$.For the **Current phase** select 0º - this will produce a cosine current profile in the coil. This current will in turn induce a relatively uniform flux density of magnitude $$B_0 = 1.46 \times 10^{-3} \, \text{T}$$ over the volume of the copper disc (radius: 3mm), placed in the center of the cylinder. To help magnetic field align vertically inside the cylinder, add **Normal Flux** boundary conditions to cylinder caps and **Tangential Flux** boundary condition to the inner face of the cylinder wall.

Boundary conditions

To help the magnetic field align vertically inside the cylinder, Normal Flux boundary condition to cylinder caps and Tangential Flux boundary condition to the inner face of the cylinder wall, should be added.

To do define the Normal Flux,

1. In the EMS manger tree right-click on the **Load/Restraint** folder and select **Normal Flux**

   .

2. Click inside the Faces for **Normal Flux box** then select Cylinder caps .
3. Click OK

To do define the Tangential Flux,

1. In the EMS manger tree right-click on the **Load/Restraint** folder and select **Tangential Flux**
.
2. Click inside the Faces for **Tangential Flux** box then select the inner face of the Cylinder wall .
3. Click OK .

Coils

To show how to define the Wound Coil, see "Force in a magnetic circuit" example.

Eddy Effects

To set up Eddy Effects, right-click the copper disc in the EMS tree and select Turn on **Eddy Effects**

.

Meshing

To get a high resolution of current density results in the copper disc, a Mesh control of 0.1 mm should be applied to the copper disc.
To do so,

1. In the EMS manger tree right-click on the **Mesh** folder and select **Apply Mesh Control**   .

2. Click inside the **Bodies** box then select the inner face of the copper disc .

3. Under Control Parameters click inside the **Element Size**   box and type 0.1 mm.

4. Click OK .

To mesh the model:

1. In the EMS manger tree, right-click on the **Mesh**   icon and select **Create Mesh**

2. Click OK .

Results

To plot the current density distribution in the copper disc, right-click the **Current Density** in the EMS tree and select **2D Plot** . For the current density **Representation** select Real (instantaneous).

Maximum current density occurs for 90° (minimum at 270º), since the eddy currents have sinusoidal time dependence (in the case of a cosine coil current and magnetic field).It is enough to select only 2 points along the disc radius – one in the center and the other one by the periphery. Type in the number of points you want in between and EMS will plot the current density along the disc radius.
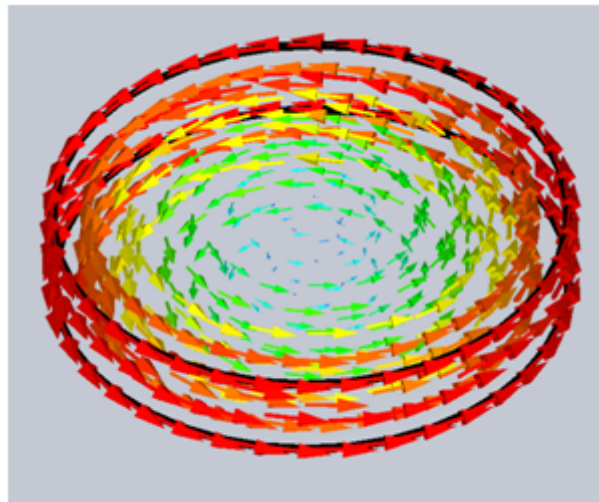

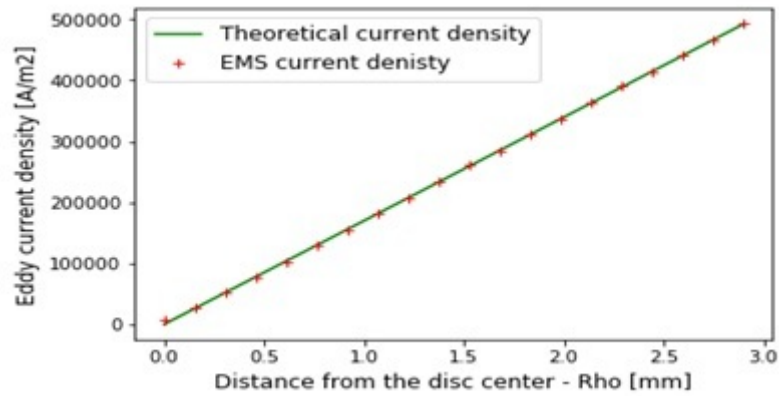***Figure 2 -*** *3D vector plot of Eddy current density inside the copper disc*

Figure 3 - *Comparison of EMS and theoretical results for eddy current density*

*The agreement between the theoretical solution (*r[m]×1.69759×108m3A?*) and the EMS 2D current density plot is displayed in Figure 3.*

Conclusion

The application note explores the behavior of eddy currents within a conducting disc under a time-varying magnetic field, providing insights through theoretical analysis and practical simulation. By examining Faraday's law and Ohm's law, it elucidates the mechanisms governing the induction and distribution of eddy currents, emphasizing their dependence on magnetic field properties and material conductivity. Through simulations in EMS, it demonstrates how to model and analyze eddy current phenomena, highlighting the importance of boundary conditions, coil setups, and meshing parameters. The note's findings offer valuable guidance for researchers and engineers seeking to understand and mitigate the effects of eddy currents in various systems, ultimately contributing to improved efficiency and reliability in electromagnetic applications. Overall, this comprehensive exploration advances the understanding and application of eddy current behavior in conducting discs, facilitating future innovations in electromagnetic engineering.