Modeling the electric field in a spherical void electrodynamic levitator

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Introduction

The spherical void electrodynamic levitator (SVEL) is a trap for individual aerosol particles. The electrodynamic balance uses an AC voltage along with a DC bias to float a suspended particle against gravity and drifting.

The SVEL we have used is an established design first proposed by Arnold and Folan in 1987 [1]. It consists of a spherical copper chamber whose walls are divided into three electrodes: two at DC biases and one with an AC voltage. These are lined with very thin insulating material. There are several small windows along its equator, and an opening at the top to act as an inlet for particle samples. The windows can be used to observe a trapped particle using a camera, as well as allow laser light through for scattering observations. Particles may be dropped in via the top inlet. This entire setup is placed

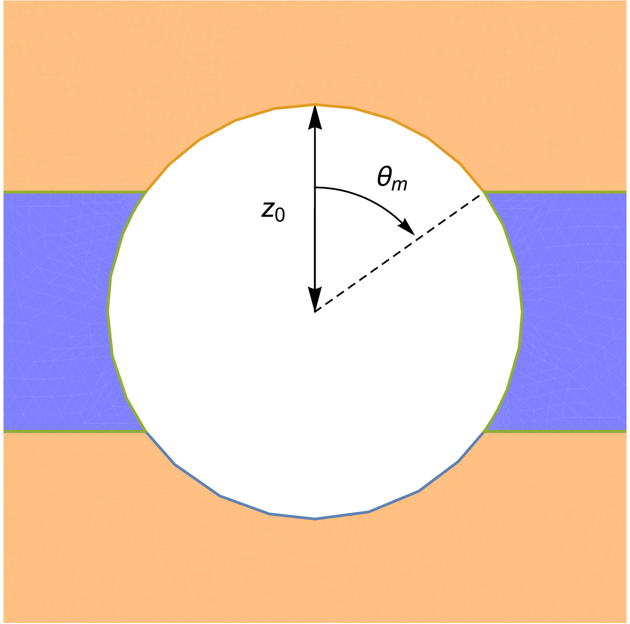


Figure 1: A cross section of the idealized SVEL. The white central region is the titular spherical void, an open space in which a particle can be trapped. Shaded regions represent the copper enclosure. The top and bottom sections of this enclosure are kept at a DC bias, and the middle acts as an AC electrode.

inside an enclosed cylinder, which can be primed with and accept a flow of atmospheric gases at different temperatures and pressures.

Figures 1 and 2 show the idealized SVEL. We will continue to refer to this "ideal" SVEL as one without the inlet or windows. Because of the simple geometry, we start with this model when characterizing the electric field. We assume the removal of some sections for these openings does not affect the field inside significantly.

The DC voltage biases are typically up to 30 volts. This bias can be tuned to support particles of particular mass-to-charge ratios, floating them vertically, against gravity. The AC voltage trapping the particle typically runs at 100 Hz and no more than ten times this rate, and has an RMS voltage of no more than 1 volt.

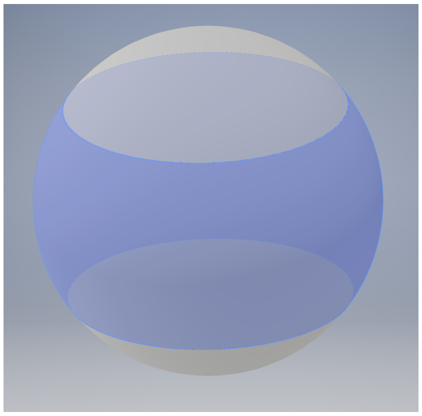


Figure 2: The SVEL in Autodesk Inventor. The EMWorks EMS add-on was used to apply voltages to the three electrode surfaces and model the resulting electric fields within.

The ideal field

Starting with a separated potential, with a DC and AC term, and noting the rotational symmetry of the geometry,

$$\Phi(r,\theta,t) = \Phi_{\mathrm{AC}}(r,\theta)\cos(\omega t) + \Phi_{\mathrm{DC}}(r,\theta),$$

the AC potential can be expressed as

$$\Phi_{\mathrm{AC}} = \sum_{n=0}^{\infty} A_n r^n P_n(\cos\theta)$$

with P_n the Legendre polynomials. The coefficients A_n are

$$A_n = [(2n+1)V_{\mathrm{AC}}/2z_0^n] \int_{-\cos\theta_m}^{\cos\theta_m} P_n(x)\ dx$$

with V_{AC} the amplitude of the AC voltage and θ_m the angle at which the upper DC electrode meets the AC electrode. Given these, the AC potential can be expressed as

$$\Phi_{\mathrm{AC}} = V_{\mathrm{AC}}\left(x_m + \sum_{n\text{ even},\ n=2}^{\infty} (r/z_0)^n \left[P_{n+1}(x_m) - P_{n-1}(x_m)\right] P_n(\cos\theta)\right)$$

with $x_m = \cos\theta_m$. The quadrupolar term, which is most critical for trapping a particle, is maximized with $x_m = 1/\sqrt{3}$. Our SVEL has been designed with this condition. We can convert this potential into an AC component of the electric field. The square of this is shown in Figure 3, representing an ideal field strength we could like to verify.

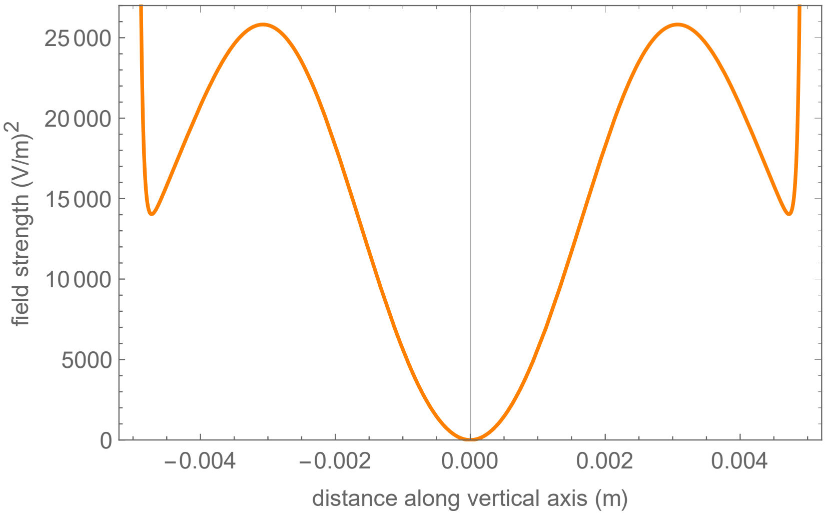


Figure 3: The ideal field strength inside the SVEL along the vertical axis, measured relative to the center.

Modeling the field

In order to confidently trap and perform experiments on charged particles, we seek to first characterize the potential and the electric field inside of the chamber, beginning with the windowless ideal.

The size of the SVEL should not influence the general shape of the result. Regardless, we have used a radius of $z_0 = 0.5$ cm. The quadrupole moment of the AC field is what primarily traps particles. With θ the angle at which the top electrode and the AC electrode meet, this portion of the field is maximized when $\cos\theta = 1/\sqrt{3}$.

The simple geometry allowed for easy creation of a model of the instrument in Autodesk Inventor and application of the voltages using EMWorks'

EMS add-on. Since the chamber size is small, and the period of the AC is long from a radiation standpoint, it is appropriate to model the oscillating electric field as a series of static fields caused by constant voltages on the AC electrode as well as the DC ones.

The top electrode was kept at 1 V, the bottom at -1 V, and the AC electrode was given six voltages representing the first quarter of a cycle, each at 1/20 the period of the cycle apart. With a maximum of 1 V, these were

$$V_{\mathrm{AC}} = \cos 0,\ \cos\left(\frac{\pi}{10}\right),\ \cos\left(\frac{\pi}{5}\right),\ \cos\left(\frac{3\pi}{10}\right),\ \cos\left(\frac{2\pi}{5}\right),\ \cos\left(\frac{\pi}{2}\right)$$

$$\approx 1,\ 0.951,\ 0.809,\ 0.588,\ 0.309,\ 0,$$

all in volts. Due to the symmetry of the instrument, we assume any results of the potential or electric field using the above are negated for corresponding negative AC voltages, and duplicated when the voltage is rising again. This way, the results from the above applied voltages can be copied to cover 20 evenly spaced snapshots of the electric field in the spherical void through a full cycle of the AC electrode.

Results

EMS produced a model of the electric field throughout the SVEL. In particular, we are interested in the field strength along the vertical axis. A 2D plot along this dimension was made from bottom to top (Figure 4). These data were exported as CSV files which could be read by other software.

In order to extract the AC component of the field, we averaged these plots over a single cycle (Figure 5) and subtracted that from each of the results. These averages end up having the same shape, with only a difference in scaling. The square of the maximum is compared to the ideal in Figure 6. The results agree well with the ideal (Figs. 7 and 8), especially in the center of the chamber, where our model of the field is critical for understanding the behavior of a trapped charged particle.

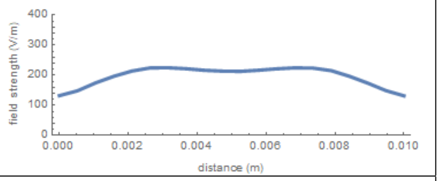
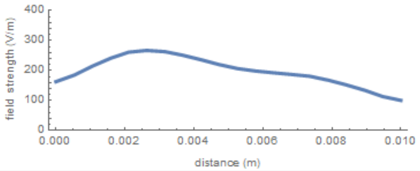
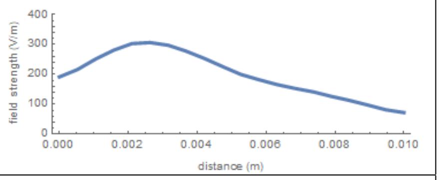
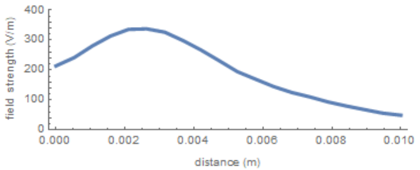
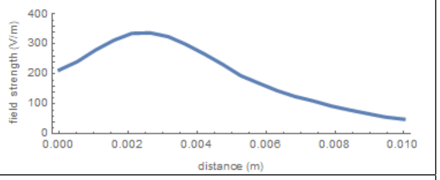
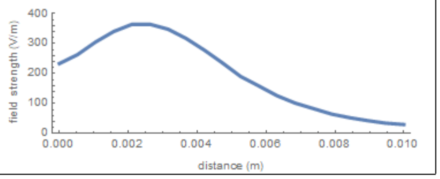
AC voltage (V)	**Fraction of period**	**field strength along z axis**
1	0/20	
0.951	1/20	
0.809	2/20	
0.588	3/20	
0.309	4/20	
0	5/20	

Figure 4: Results from the EMWorks EMS model of the SVEL's electric field strength from lower to upper DC electrodes. Duplicates and mirror images of these are used for the following 3/4 of the cycle.

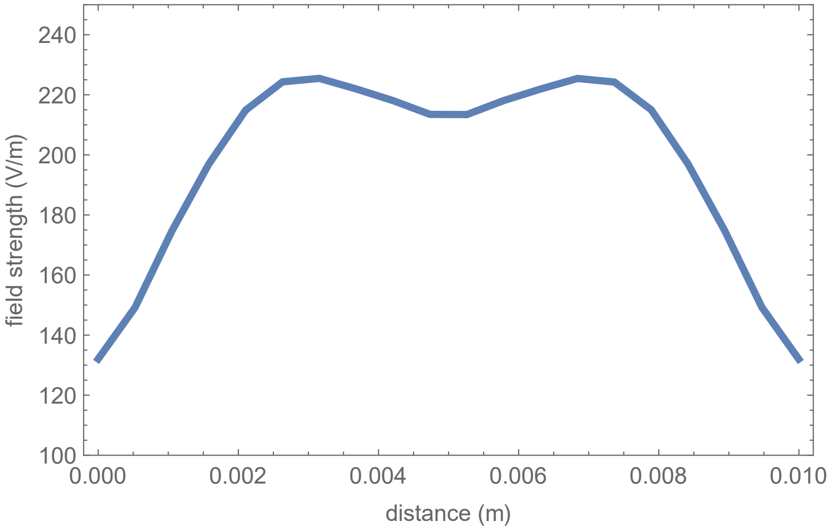


Figure 5: The mean electric field strength found by averaging the results.

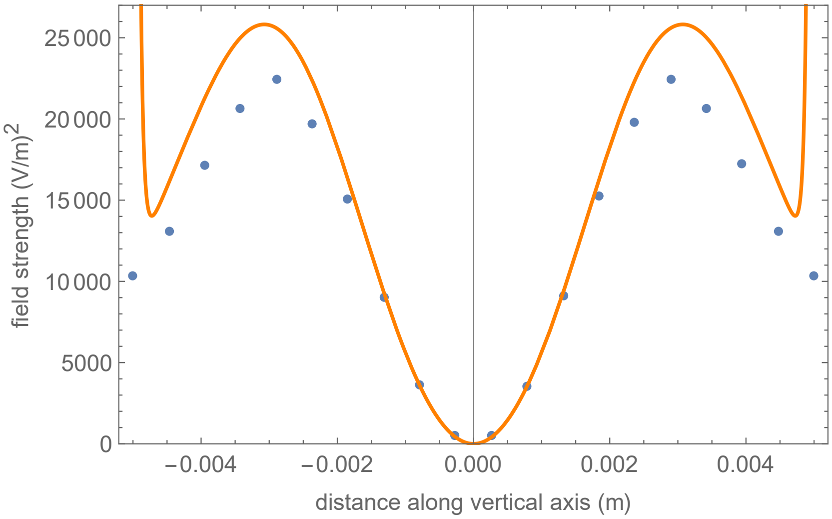


Figure 6: The maximum average result from EMS (blue points) compared to the ideal analytic solution (orange). Note that this is plotted relative to the center of the chamber, not the bottom.

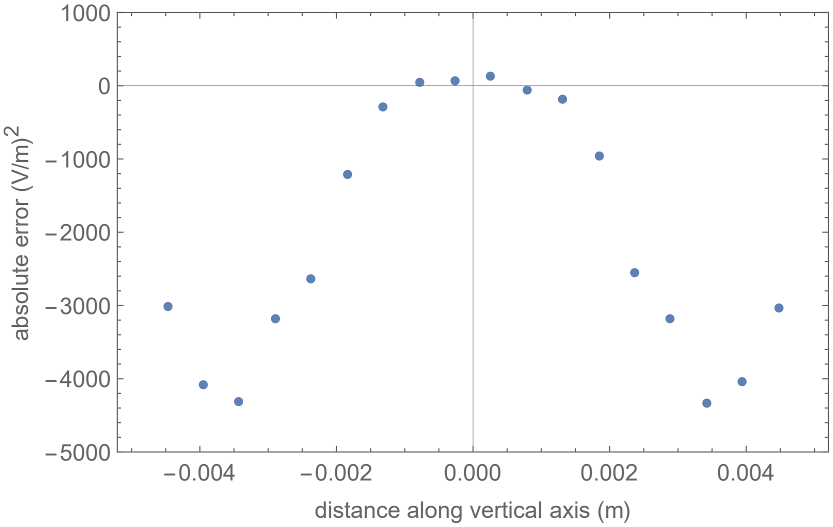


Figure 7: The absolute error between the ideal analytic solution and the EMS result.

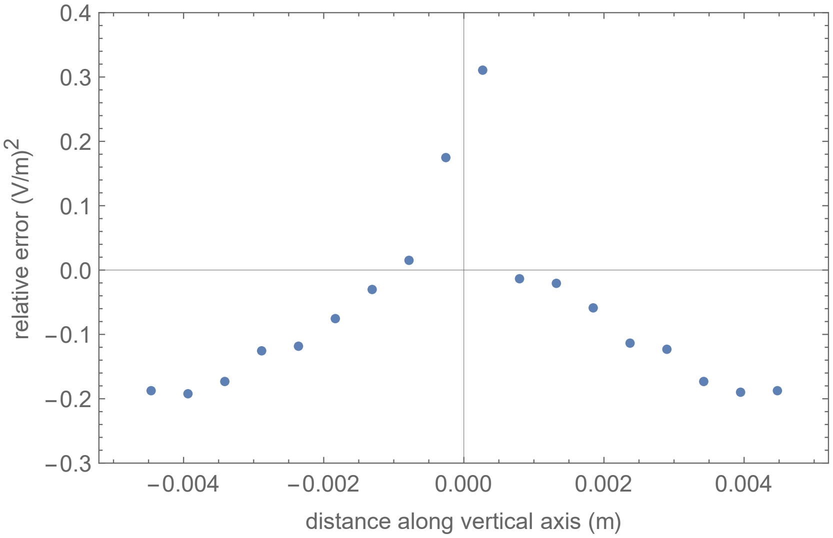


Figure 8: The relative error between the ideal and the EMS result.

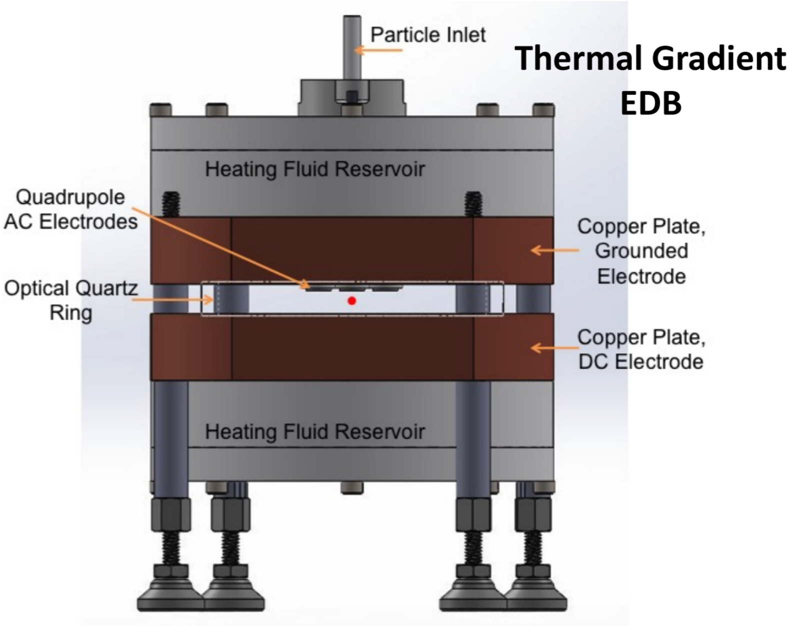


Figure 9: The advanced electrodynamic balance featuring four top-mounted electrodes.

This analysis has given us some confidence in our understanding of the works of the electrodynamic balance. Going forward, we would like to make several refinements to this model. First, we plan to produce higher resolution plots of the AC potential and the electric field along the z-axis, each spaced at shorter time periods. Continuing to use the ideal SVEL, we would like to produce a function of the field throughout the volume of the chamber. This model will help us to understand the behavior of charged particles as they orbit the most stable point in the trap. We would then like to continue using the EMS software with an updated SVEL model — one including the viewing windows and top inlet — to see the influence of these features.

Ulimately, we would like to apply these same techniques to another electrodynamic levitator (Figure 9). This advanced balance uses four “button” AC electrodes mounted to the top of a cylindrical chamber to generate the quadrupolar trapping field.

References

[1] S. Arnold and L.M. Folan. Spherical void electrodynamic levitator.

[2] W.H. Hartung and C.T. Avedisian. The electrodynamic balance, November 1990.

[3] W.H. Hartung and C.T. Avedisian. On the electrodynamic balance. *Proceedings: Mathematical and Physical Sciences*, 437(1900), May 1992.